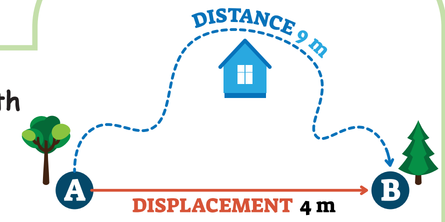
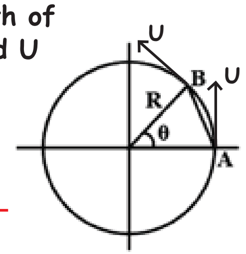
MOTION ALONG A STRAIGHT LINE

- Distance = Length of actual path
- Displacement = Length of shortest path
- Distance $\geq$ |displacement|



A particle moves from A to B in a circular path of radius R covering an angle θ with uniform speed U



- Distance = $R\theta$
- Displacement = $2R\sin\left(\frac{\theta}{2}\right)$
- Ratio of Displacement to Distance $= \dfrac{2\sin\left(\frac{\theta}{2}\right)}{\theta}$
- Time $t = \dfrac{R\theta}{U}$
- Average Velocity $= \dfrac{2U\sin\left(\frac{\theta}{2}\right)}{\theta}$
- Average Acceleration $= \dfrac{2U^2\sin\left(\frac{\theta}{2}\right)}{R\theta}$

- **For uniform motion**

Displacement = velocity × time

Average speed = |average velocity|=|instantaneous velocity|

• Time average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Totaltime elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

If $t_1 = t_2 = t_3 = \dots = t_n$

then

$$v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n} = \frac{v_1 + v_2}{2} \quad \text{(Arithmetic mean of speeds)}$$

• Distance average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Totaltime elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}$$

If $s_1 = s_2 = s_3 = \dots s_n s$

then

$$v_{av} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}} = \frac{2V_1V_2}{V_1 + V_2} \quad \text{(Harmonic mean of speeds)}$$

Instantaneous Velocity $v = \dfrac{dx}{dt}$ $\qquad \Delta x = \int v dt$

Instantaneous Acceleration $a = \dfrac{dv}{dt}$ $\qquad \Delta v = \int a dt$

Case 1	Case 2	Case 3
V or x = f(t)	V = f(x)	t = f(x) then
$a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	$a = V\dfrac{dV}{dx}$	a = −(double diff. of t w.r.to x) × V^3

Displacement $\xrightarrow{\text{Differentiation}}$ Velocity $\xrightarrow{\text{Differentiation}}$ Acceleration

Acceleration $\xrightarrow{\text{Integration}}$ Velocity $\xrightarrow{\text{Integration}}$ Displacement

Motion with constant acceleration: Equations of motion

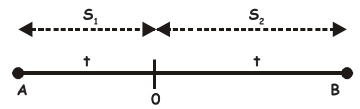
(i) $v = u + at$

(ii) $S = ut + \frac{1}{2}at^2$

- A Person travels from A to B covers unequal distances in equal interval of time with constant acceleration a then

initial velocity $U = \dfrac{3S_1 - S_2}{2t}$

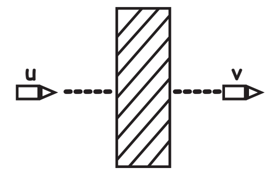
Acceleration $a = \dfrac{S_2 - S_1}{t^2}$



(iii) $v^2 = u^2 + 2a.s$

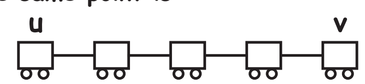
- The number of planks required to stop the bullet

$N = \dfrac{u^2}{u^2 - v^2}$

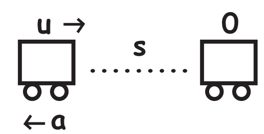


- The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

$v_{Mid} = \sqrt{\dfrac{u^2 + v^2}{2}}$



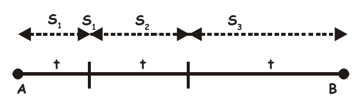
- Calculation of stoping distance $s = \dfrac{u^2}{2a}$



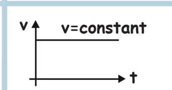
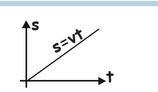
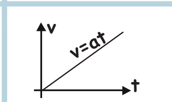
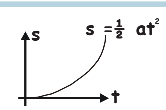
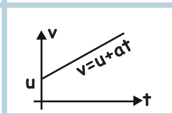
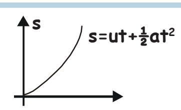
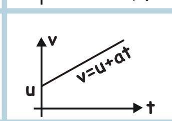
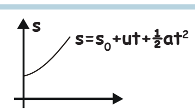
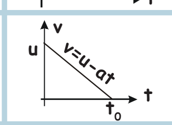
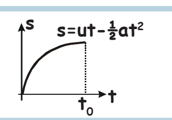
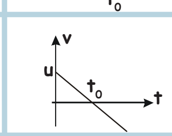
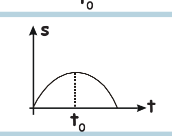
(iv) $S_n = u + \frac{a}{2}(2n - 1)$

- Ratio of distance travelled in equal interval of time in a uniformly accelerated motion from rest

$S_1 : S_2 : S_3 = 1:3:5$



- for uniform accelerated motion $v_{avg} = \dfrac{u + v}{2}$

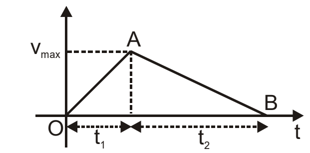
Different Cases	v-t graph	s-t graph
1. Uniform motion		
2. Uniformly accelerated motion with u = 0 at t=0		
3. Uniformly accelerated with u ≠ 0 at t=0		
4. Uniformly accelerated motion with u≠0 and s=s₀ at t=0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve $\left[v = \dfrac{dx}{dt}\right]$
- v-t curve area gives displacement. $[\Delta x = \int v dt]$
- Slope of velocity-time curve = instantaneous acceleration $\left[a = \dfrac{dv}{dt}\right]$
- a-t curve area gives change in velocity. $[\Delta v = \int a dt]$

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t, the maximum velocity attained

$V_{max} = \dfrac{\alpha\beta}{\alpha + \beta}t \qquad$ Total Distance $= \dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha + \beta}\right)t^2$



MOTION UNDER GRAVITY

Sign Convension

(i) initial velocity

+ve = upward motion

-ve = downward motion

(ii) Acceleration

Always -ve

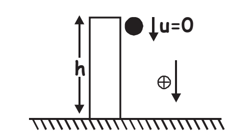
(iii) Displacement

+ve = final position is above initial position

-ve = final position is below initial position

Zero = final position & initial position are at same level

- Object is dropped from top of a tower



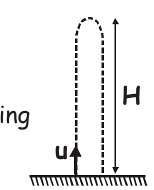
(i) Ratio of displacement in equal interval of time $S_1 : S_2 : S_3 \dots = 1:3:5\dots$

(ii) Ratio of time of covering equal distance

$t_1 : (t_2 - t_1) : (t_3 - t_2) : \dots : (t_n - t_{n-1}) = 1 : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) : \dots : (\sqrt{n} - \sqrt{n-1})$

(iii) Ratio of distance covered at the end of time $t : 2t : 3t : \dots = 1^2 : 2^2 : 3^2 \dots$

- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then



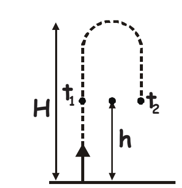
(i) Maximum height attained $H = \dfrac{u^2}{2g}$

(ii) Time of ascent = time of descent $\dfrac{u}{g}$

(iii) Total time of flight $= \dfrac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

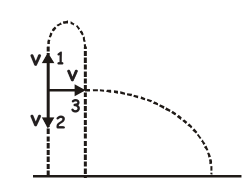
- At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then



height of point $h = \frac{1}{2}gt_1t_2 \qquad$ Maximum height $H = \frac{1}{8}g(t_1 + t_2)^2$

$t_1 + t_2 = \dfrac{2u}{g}$

- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then



$t_3 = \sqrt{t_1t_2}$ & height from where the particle was throw is $h = \frac{1}{2}gt_1t_2$